Math 2471 – Spring 2010 Name Todd

Review for Test 3

For full credit: use calculus to solve problems, circle answers, and **show all your work**.

1) Use the limit process to find the area under the curve of $y = x^2 + 2$ on $[2, 5]$. $\rightarrow \frac{3}{n}$ is width

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}+2\right)^2+2\right]\left(\frac{3}{n}\right)$

$= \ldots$

See side board in 233.

2) I evaluated the integral $\int_{-2}^{2}(x^3)dx =$ and found the result to be zero. I double checked my work and found no errors; however, I know there is some area between the graph and the x-axis. Please explain the result. It's an odd function so it has the same amount of area above (+) and below (−) the x-axis.

3) Evaluate the integral:

$\int_2^5(-3v+4)dv = -\frac{3}{2}v^2+4v+c\Big|_2^5$

$= \left(-\frac{3}{2}\cdot 25+4\cdot 5+c\right)-\left(-\frac{3}{2}\cdot 4+4\cdot 2+c\right)$

$= -\frac{75}{2} + 20 + c + \frac{12}{2} - 8 - c$

$= -\frac{63}{2} + 12$

$= -19.5$ ("−" b/c below x-axis)

4) Determine the area under the curve $y = (3-x)\sqrt{x}$ between $x = 4$ and $x = 9$.

$\int_4^9 3x^{1/2} - x^{3/2}\,dx$

$= 2x^{3/2} - \frac{2}{5}x^{5/2} + C\Big|_4^9$

$= 2(9^{3/2}) - \frac{2}{5}(9)^{5/2} + C - \left(2(4^{3/2}) - \frac{2}{5}(4^{5/2}) + C\right)$

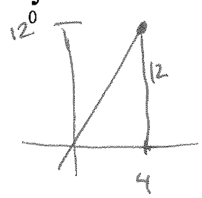
$= 2\cdot 27 - \frac{2}{5}\cdot 243 + C - \left(2\cdot 8 - \frac{2}{5}\cdot 32 + C\right)$

$= 54 - 97.2 - 16 + 12.8$

$= -46.4$ sq. un. area > 0 ∴ 46.4 sq. un

5) Evaluate the integral without using calculus or your calculator:

$\int_0^4 3x\,dx =$



$A = \frac{1}{2}\cdot b\cdot h$

$= \frac{1}{2}\cdot 4\cdot 12$

$= 24$ sq. un.

6) Find the indefinite integral and check the result by differentiation of $\int(t^2 - \sin t)dt$.

$f(t) = \frac{1}{3}t^3 + \cos t + C$

Check: $f'(t) = t^2 - \sin t$ ✓

7) Evaluate the integral: $\int_0^1(x - x^2)dx =$.

$= \frac{1}{2}x^2 - \frac{1}{3}x^3 + C\Big|_0^1$

$= \frac{1}{2}\cdot 1 - \frac{1}{3}\cdot 1 + C - (0 - 0 + C)$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

8) Evaluate the integral: $\int_{-1}^{1}(t^3 - 9t)dt =$

$= \frac{1}{4}t^4 - \frac{9}{2}t^2 + C\Big|_{-1}^{1}$

$= \frac{1}{4}\cdot 1 - \frac{9}{2}\cdot 1 + C - \left(\frac{1}{4} - \frac{9}{2} + C\right)$

$= \frac{1}{4} - \frac{9}{2} - \frac{1}{4} + \frac{9}{2}$

$= 0$ (b/c it's an odd function – same amount above and below x-axis.)

9) Find the indefinite integral: Let $u = 1-x^2$, $du = -2x\,dx$

$\int 5x * \sqrt[3]{1-x^2}\,dx = -\frac{5}{2}\int u^{1/3}du$

$= -\frac{5}{2}\cdot\frac{3}{4}u^{4/3} + C$

$= -\frac{15}{8}(1-x^2)^{4/3} + C$

10) Evaluate the integral: $\int_0^{\pi}(1 + \sin x)dx =$.

$= x - \cos x + C\Big|_0^{\pi}$

$= \pi - \cos\pi + C - (0 - \cos 0 + C)$

$= \pi - (-1) + C - (0 - (+1) + C)$

$= \pi + 2$ $\pi + 1 - -1$

Need slope:
(aka Derivative)
$y' = \frac{1}{x^3}\cdot 3x^2$

11) Find the equation of a line tangent to: $y = \ln x^3$ at the point (1,0).

Slope @ $x=1$: $\frac{1}{1^3}\cdot 3\cdot 1^2 = 3 \therefore m=3$

$y = mx+b$

$y = 3x+b$

$0 = 3(1)+b$

$-3 = b$

$\therefore y = 3x + -3$

12) Find the derivative of: $y = \ln(x\sqrt{x^2-1})$. $(x^2-1)^{1/2}$

$= \ln x + \frac{1}{2}\ln(x^2-1)$

$y' = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2-1}$

$y' = 1/x + \frac{x}{x^2-1}$

13) If $g(x) = (\ln x)^4$ then find $g'(x) =$

$g'(x) = 4(\ln x)^3\cdot\frac{1}{x}$ (Chain rule)

$= \frac{4(\ln x)^3}{x}$

14) Find: $\frac{1}{2}\int\frac{2x}{x^2+1}dx =$

Let $u = x^2+1$
$du = 2x\,dx$

$= \frac{1}{2}\int\frac{1}{u}\,du$

$= \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln(x^2+1) + C$

15) Find the derivative of: $y = \ln|\sin x|$.

$u = \sin x$

$du = \cos x$

$= \frac{\cos x}{\sin x}$

$= \cot x$

16) Evaluate: $\frac{5}{3}\int_0^4\frac{\cancel{5}\,3}{3x+1}dx =$

$u = 3x+1$
$du = 3\,dx$

$= \frac{5}{3}\int_0^4\frac{1}{u}\,du$

$= \frac{5}{3}\ln|3x+1|\Big|_0^4$

$= \frac{5}{3}\ln|13| - \frac{5}{3}\ln 1$

≈ 4.27

17) Use logarithmic differentiation to find the derivative of: $y = x^{x-1}$

$f = x-1 \quad f' = 1$

$g = \ln x \quad g' = 1/x$

$\ln y = (x-1)\ln x$

Derive: $\frac{1}{y}\,dy/dx = (\ln x + 1/x(x-1))$

$dy/dx = x^{x-1}\left(\ln x + \frac{x-1}{x}\right)$

18) Find the integral: $-1\int 5^{-x}\cancel{-1}\,dx =$

Let $u = -x$
$du = -1\,dx$

$= -1\int 5^u\,du$

$= \frac{-1\cdot 5^u}{\ln 5} + C$

$= \frac{-5^{-x}}{\ln 5} + C$

19) Solve the equation for x:

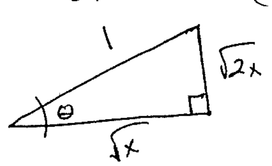
$\sin(\arcsin\sqrt{2x}) = \sin(\arccos\sqrt{x})$

Graph - Calc - Intersect:

$x = 1/3$

$\sqrt{2x} = \sin(\arccos\sqrt{x})$

$2x = \sin^2(\arccos\sqrt{x})$



$\sqrt{x}^2 + \sqrt{2x}^2 = 1^2$

$x + 2x = 1$

$3x/3 = 1/3$

$x = 1/3$

20) Solve the differential equation: $\frac{dy}{dx} = x+2$

$y = \frac{1}{2}x^2 + 2x + C$

Higher on [1, 2]

Higher function on [0, 1] ∴ more area under

21) Solve the differential equation:
$\frac{dy}{dx} = y + 2$ $\Rightarrow \frac{dy}{y+2} = dx$

$\int \frac{dy}{y+2} = \int dx$

$\ln|y+2| = x + C_1$

$y + 2 = e^{x+C_1}$

$y = e^{x+C} - 2$

$y = C_1 e^{x} - 2$

22) Find the **area** between $f(x) = (x-1)^3$ and $g(x) = x - 1$ on [0 , 2].

$= -\int_1^2 (x-1)^3 dx + \int_1^2 x - 1\, dx + \left(\int_0^1 x-1\, dx - \int_0^1 (x-1)^3 dx\right)$

$= -\int_1^2 x^3 - 3x^2 + 3x - 1\, dx + \int_1^2 x - 1\, dx + \int_0^1 x - 1\, dx + \int_0^1 x^3 - 3x^2 + 3x - 1\, dx$

$= (\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x\Big|) + \frac{1}{2}x^2 - x\Big| + (\frac{1}{2}x^2 - x\Big|_0^1) + (\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x)\Big|$

$= .25 + .5 + (+\frac{1}{2} + -.25)$

$= .25 + .25 = \frac{1}{2}$ sq. un.

23) Solve the differential equation:
$y' = \frac{5x}{y}$

$y \cdot y' = 5x$

$\int y \cdot y' dx = \int 5x\, dx$

$\int y\, dy = \int 5x\, dx$

$\frac{1}{2}y^2 + C_1 = \frac{5}{2}x^2 + C_2$

$y^2 - 5x^2 = C$

24) Find the derivative of $y = \ln|x|$.

$= 1/x$

25) Find the integral: $\int 5e^{5x} dx =$

Let $u = 5x$
$du = 5dx$

$= \int e^u du$

$= e^u + C$

$= e^{5x} + C$

26) If $f(x) = \sqrt{x-4}$ then find $f^{-1}(x) =$

Derivative:
$f(x) = (x-4)^{1/2}$

$f'(x) = \frac{1}{2}(x-4)^{-1/2} \cdot 1$

$f'(x) = \frac{1}{2\sqrt{x-4}}$

Inverse:
$x = \sqrt{y-4}$

$x^2 = y - 4$

$x^2 + 4 = y$

$f^{-1}(x) = x^2 + 4$

27) Solve for x: $e^{\ln x} = 4$

$\ln x \cdot \ln e = \ln 4$

$\ln x = \ln 4$

$x = 4$

28) Use logarithmic differentiation to find the derivative of: $y = x^{2/x}$

Derive: $\ln y = 2/x \ln x$

$f = 2x^{-1}$ $f' = -2x^{-2}$

$g = \ln x$ $g' = 1/x$

$\frac{1}{y}\, dy/dx = \frac{-2\ln x}{x^2} + \frac{2}{x^2}$

$dy/dx = y\left(\frac{2}{x^2} + \frac{-2\ln x}{x^2}\right)$

$dy/dx = x^{2/x}\left(\frac{2 - 2\ln x}{x^2}\right) = 2x^{2/x - 2}(\cancel{1-\ln x})$

$(1 - \ln x)$

29) Find the derivative of:
$f(x) = 2\arcsin(x-1)$

$f'(x) = \frac{2}{\sqrt{1-(x-1)^2}}$

$f'(x) = \frac{2}{\sqrt{-x^2+2x}}$

30) Evaluate the integral:
$\int_0^{1/6} \frac{3}{\sqrt{1-9x^2}} dx = \int_0^{1/6} \frac{1}{\sqrt{1-(3x)^2}} 3\,dx$

Let $u = 3x$
$du = 3dx$

$a = 1$

$= \arcsin \frac{3x}{1} + C \Big|_0^{1/6}$

$= \arcsin 3(1/6) + C - (\arcsin 3(0) + C)$

$= \arcsin 1/2 - \arcsin 0$

$= \pi/6 - 0 = \pi/6$

OR

$30° - 0° = 30°$